A mathematics challenge to try: Comments & Solutions

Comments

How did you do on the five questions? This is what happened in the study which involved 69 pre-service teachers. The average score on the entire test was 49%. About 70% of the scores were in the range from 37% to 61%. Do the results mean that the pre-service teachers who took the test are “not smart” in mathematics? Not likely.

More likely, the low scores indicate the mathematics instruction that the pre-service teachers themselves experienced when they were in the elementary (and senior) grades. The test results probably reflect learning experiences that did not engage them in thinking about mathematical concepts, principles, and explanations that underlie procedures and notational systems. Their instruction in mathematics likely was one of “*You do this; you do that - don’t ask why*.” A folk saying about learning fraction division says it well. “*Yours is not to reason why, just invert and multiply*.” When the pre-service teachers in the study were asked to think back on their own mathematical learning experiences they overwhelmingly described the kind of instruction mentioned above. There were a few exceptions for a special teacher they had here and there.

If the reader had difficulty with the five questions, perhaps you received the same kind of mathematics instruction as did the pre-service teachers in the study. But that was a while ago. Surely, the quality of mathematics instruction is not like that any more. While progress is being made in improving the quality of instruction in Manitoba, unfortunately, it is somewhat slow and haphazard progress.

You can investigate that for yourself. Next time you are in an elementary classroom for a practicum experience, observe how often mathematics instruction occurs, when it occurs, and the way it occurs. Ask yourself the following kinds of questions about what you observe.

- Is mathematics instruction a regular event in the classroom or does it happen occasionally? Does mathematics instruction occur mostly at the end of the day, close to recess time, mostly at calendar time, and/or at “slack” time?
- Does mathematics instruction motivate children to want to learn about mathematics or is it mostly a chore or neutral event for them? Does it consist mostly of worksheets and assigned pages from text books? Is learning connected to something meaningful? Is risk-taking encouraged? Do the mathematical learning activities challenge children or do the activities seem more like busy work?
- Do the activities have a clear mathematical purpose or is their purpose something like “*Children will have fun learning about geometry by doing the activity*.” Is seeking evidence of learning a regular feature of instruction? Does new learning happen or do children mostly relearn old knowledge?
- Do children relate to manipulative-based activities as a good way to learn mathematics or as an opportunity to fool around? Do they see any mathematical purpose underlying manipulative-based activities?

Solutions

Question #1

The average score was 31%. Most of the participants did not attempt it. Of the small number who tried the question, most figured out a solution.

Here is the one solution. Start the two egg timers together. Do not begin to cook the egg yet. When the 7-minute timer is done, there are four minutes left on the 11-minute timer. Begin to cook the egg now. When the 11-minute timer is done, flip it over. Continue cooking the egg until the 11-minute timer is done.

Question #2

The average score was 78% for the first part of the question. Most participants were able to determine that Tony should give Mary $80 to equalize contributions. That way, Tony spent 35 + 80 = 115 and Mary spent 195 - 80 = 115. Those who did not do the question correctly seem to have misread it or made arithmetic errors.

Many of the participants did not attempt the second question. Only 20% of the them were able to translate what they did with the actual numbers into a general formula that could be used for all such circumstances. Those who provided an incorrect formula made errors related to the order of operations, the role of brackets, and/or the use of symbols. No one expressed the general solution using a statement such as “The amount needed to adjust for an imbalance in contributions can be obtained by determining the difference in the amounts spent divided by 2.”

Question #3

The average score for this question was 25%. Most of the participants who did not do well on it selected 'a) .45' or 'b) 45' as the remainder. That choice indicates a poor understanding of decimal representations and remainders. It suggests that they think that the decimal part of a number is equivalent to the remainder.

The correct answer is ‘c) 144’. The easiest way to obtain it is to work out the answer to .45 x 320. The reasoning behind that involves the inverse (do/undo) relationship between division and multiplication. Calculating 27642064 ÷ 320 results in a whole number part and a remainder. The whole number form of the remainder is changed into decimal form by dividing it by 320. Changing the decimal form of the remainder back into the whole number form involves reversing the arithmetic by doing ‘decimal form x 320’.

Question #4

The average score was 45%. Most of the participants who did not do well on it tried to use a vaguely-remembered formula about arithmetic series that they had been taught in a grade 12 mathematics course. The formula did not apply to the question. Others who did not do well failed to detect a pattern in the numbers or the arrangement of dots.

Some did notice that from 2 to 6 is an increase of 4, from 6 to 12 is an increase of 6, and from 12 to 20 is an increase of 8. Fewer realized the general pattern underlying that - an increase of 2 each time. Some who did see that general pattern worked out the solution by following the pattern to the 50th term. Others made arithmetic errors along the way or could not keep track of the terms.

Only a few people saw a geometric pattern in the dots. The first term is a 1 x 2 array; the second term is a 2 x 3 array; the third term is a 3 x 4 array; the fourth term is a 4 x 5 array. The 50th term would be a 50 x 51 array with 50 x 51 = 2550 dots.

Question #5

The average score for this question was 25%. Most people picked; 'e) must be done first because it is a mathematical law'. That may indicate a high level of "brainwashing" during their schooling experiences. In other words, they may have been given little opportunity or incentive to think about or ask questions about mathematical notations. Rather, they may have been told that brackets must be done first and that's the end of the matter. If so, it is easy to understand why someone would think that there is a mathematical law that brackets must be done first.

The best choice is 'a) should be done whenever it is convenient'. Brackets are not "do me first" symbols. The following is a simple example to show that.

Consider the arithmetic expression; '2 x 3 + 5 x (4 + 6)'. The brackets do not have to be done first. In fact, they do not have to be done at all in the manner that many of the participants thought (doing 4 + 6). Here is one way to calculate the result that does not involve adding 4 and 6.

First do 2 x 3, getting 6. Now we have '6 + 5 x (4 + 6)'. Next do 5 x 4 and 5 x 6 (the distributive principle in action) getting 20 + 30. Now we have '6 + 20 + 30'. Finally add these numbers getting 56 as the final result.

<u>Brackets are containers</u> that have the potential to alter the structure of an arithmetical/algebraic expression. They may or may not change the structure - it depends. Two examples follow.

Consider the simple expression: '2 x 3 + 5'. Brackets can be placed in it in a variety of ways. This is one way; '(2 x 3) + 5'. In that case, the structure of the expression has not changed. It is still, for example, 2 groups of 3 peanuts put together with 5 peanuts.

However, when brackets are placed in the expression '2 x 3' + 5' in this way, '2 x (3 + 5)', the structure changes. Now the expression means 2 groups of 3 peanuts put together with 2 groups of 5 peanuts or it means 2 groups of 8 peanuts. Either meaning leads to a result of 16 peanuts.